MATHEMATICS 9th SCIENCE GROUP

7/18/2020

Chapter 3.

LOGARITHM

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Contents

Scientific Notation:
A number written in the form $a \times 10^n$, $where\ 1 \le a \le 10$ and n is an integer, is called the scientific notation.

Example:
Write each of the following ordinary numbers in scientific notation.

Solution:
(i) 30600= 3.06×10^4
$(move\ decimal\ point\ four\ places\ to\ the\ left)$
(ii) $0.000058 = 5.8 \times 10^{-5}$
$(move\ dicemal\ point\ five\ places\ to\ the\ right)$

Example:
Change each of the following numbers from scientific notation to ordinary notation.

Solution:
(i) $6.35 \times 10^6 = 6350000$
$(Move\ the\ decimal\ point\ six\ places\ to\ the\ right)$
(ii) $7.61 \times 10^{-4} = 0.000761$
$(Move\ the\ decimal\ point\ four\ places\ to\ the\ left)$

Exercise 3.1

Question.1. Express each of the following numbers in scientific notation.

(i). 5700
Solution.
$5700 = 5 \wedge 700.$
$$5700 = 5.7 \times 10^3$$
Answer.

(ii). $49,800,000$
Solution.
$49,800,000 = 4 \wedge 9,800,000.$
$$49,800,000 = 4.98 \times 10^7$$
Answer.

(iii). $96,000,000$
Solution.
$96,000,000 = 9 \wedge 6,000,000.$
$$96,000,000 = 9.6 \times 10^7$$
Answer.

(iv). 416.9
Solution.
$416.9 = 4 \wedge 16.9$
$$416.9 = 4.169 \times 10^2$$
Answer.

(v). $83,000$
Solution.
$83,000 = 8 \wedge 3,000.$
$$83,000 = 8.3 \times 10^4$$
Answer.

(vi). 0.00643
Solution.
$0.00643 = 0.006 \wedge 43$
$$0.00643 = 6.43 \times 10^{-3}$$
Answer.

(vii). 0.0074
Solution.
$0.0074 = 0.007 \wedge 4$
$$0.0074 = 7.4 \times 10^{-3}$$
Answer.

(viii). $60,000,000$
Solution.
$60,000,000 = 6 \wedge 0,000,000.$
$$60,000,000 = 6.0 \times 10^7$$
Answer.

(ix). 0.00000000395
Solution.
$0.00000000395 = 0.000000003 \wedge 95$
$$0.00000000395 = 3.95 \times 10^{-9}$$
Answer.

(x). $\frac{275,000}{0.0025}$
Solution.
$\frac{275,000}{0.0025} = \frac{2\wedge 75000.}{0.002\wedge 5}$

$$\frac{275,000}{0.0025} = \frac{2.75 \times 10^5}{2.5 \times 10^{-3}}$$
$$= \frac{2.75}{2.5} \times 10^5 \times 10^3$$
$$= 1.1 \times 10^{5+3}$$
$$= 1.1 \times 10^8$$
Answer.

Question.2. Express the following numbers in ordinary notation.

(i). 6×10^{-4}
Solution.
$$6 \times 10^{-4} = \frac{6}{10^4}$$
$$= \frac{6}{10000}$$
$$= 0.0006$$
Answer.

(ii). 5.06×10^{10}
Solution.
$$5.06 \times 10^{10} = 5.06 \times 10,000,000,000$$
$$= 50,600,000,000.$$
Answer.

(iii). 9.018×10^{-6}
Solution.
$$9.018 \times 10^{-6} = \frac{9.018}{10^6}$$
$$= \frac{9.018}{1,000,000}$$
$$= 0.000009018$$
Answer.

(iv). 7.865×10^8
Solution.

$$7.865 \times 10^8 = 7.865 \times 100,000,000$$
$$= 786,500,000.$$

Answer.

Logarithm of real numbers:

$if\ a^x = y\ then\ x\ is\ called\ the\ logarithm\ of\ y\ to\ the\ base\ \text{"a"}\ and\ its\ written\ as\ \log_a y = x\ \ where\ a > 0, a \neq 1\ and\ y > 0$

$i.e\ the\ logarithm\ of\ a\ number\ y\ to\ the\ base\ \text{"a"}\ is\ the\ index\ x\ of\ the\ power\ to\ which\ a\ must\ be\ raised\ to\ get\ that\ number\ y.$

$the\ relation\ a^x = y\ and\ \log_a y = x\ are\ equivalen$

When one relation is given, it can be converted into the other. Thus

$$a^x = y \iff \log_a y = x$$

Example: find $log_4 2$ $\ i.e.\ find\ log\ of\ 2\ to\ the$ Base 4.

Solution:

Let $\log_4 2 = x$

$$then\ its\ exponenenial\ form\ is\ 4^x = 2$$
$$i.e\ \ 2^{2x} = 2^1 \Rightarrow 2x = 1$$
$$\because x = \frac{1}{2} \Rightarrow log_4 2 = \frac{1}{2}$$

Deductions from Definition of logarithm

1. Since $a^0 = 1,\ log_a 1 = 0$
2. $Since\ a^1 = a\ , log_a\ a = 1$

Common logarithm:

If the base of logarithm is taken as 10 then logarithm is called Common Logarithm.

Characteristic:

The integral part of the logarithm of any number is called the characteristic.

Mantissa: the fractional part of the logarithm of a number is called the mantissa. Mantissa is always positive.

Example: find the mantissa of the logarithm of 43.254

Solution:

Rounding off 43.254 $we\ consider\ only\ the\ four$ Significant digits4325.

(i) We first locate the row corresponding to 43 in the log tables and

(ii) Proceed horizontally till we reach the column corresponding to 2. The number at the intersection is 6355.

(iii) Again proceeding horizontally till the mean difference column corresponding to 5 intersects this row. We get the number 5 at the intersection.

(iv) Adding the two numbers 6355 and 5 we get .6360 as the mantissa of the logarithm of 43.25

Example:

Find the mantissa of the logarithm of 0.002347

Solution:

Here also, we consider only the four Signiant digits 2347

We first locate the row corresponding to 23 in the logarithm tables and proceeding to 4 the resulting number 3692. The number at the intersection of this row and the mean difference column corresponding to 7 is 13. Hence the sum of 3692 and 13 gives the mantissa of the logarithm of 0.0023476 as o.3705

Example:

1. **Find log 278.23**
2. **Log0.07058**

Solution:

1. $278.22\ can\ be\ rounded\ off\ as\ 278.22$

The characteristic is 2 and the mantissa, using log tables, is .4443

$$\because log278.23 = 2.4443$$

2. The characteristic of log$o.07058\ is - 2$ which is written as $\bar{2}$ by convention.

Using log tables the mantissa is .8487, so that

$$log0.07058 = \bar{2}.8487$$

Example:

Find the numbers whose logarithms are

(i) 1.3247

(ii) $\bar{2}.1324$

Solution:

(i) 1.3247

antilog$1.3247 = 21.12$

(ii) $\bar{2}.1324$

$$antilog(\bar{2}.1324)\ \ is\ \ 0.01356$$

Example 3.2

Question.1. Find the common logarithm of the following numbers:

(i). 232.92

Solution.

$$Characteristics = 2$$
$$Mantisa = 0.3672$$
$$Log(232.92) = 2.3672$$

Answer.

(ii). 29.326

Solution.

$$Characteristics = 1$$
$$Mantisa = 0.4672$$
$$Log(29.326) = 1.4672$$

Answer.

(iii). 0.00032

Solution.

$$Characteristics = -4$$
$$Mantisa = 0.5051$$
$$Log(0.00032) = \bar{4}.5051$$

Answer.

(iv). 0.3206

Solution.

$$Characteristics = -1$$
$$Mantisa = 0.5059$$
$$Log(0.3206) = \bar{1}.5059$$

Answer.

Question.2. If $log31.09 = 1.4926$, find the values of the following

(i). $log3.109$

Solution.

$$Characteristics = 0$$
$$Mantisa = 0.4926$$
$$Log(3.109) = 0.4926$$

Answer.

(ii). $log310.9$

Solution.

$$Characteristics = 2$$
$$Mantisa = 0.4926$$
$$Log(310.9) = 2.4926$$

Answer.

(iii). $log0.003109$

Solution.

$$Characteristics = -3$$
$$Mantisa = 0.4926$$
$$Log(0.003109) = \bar{3}.4926$$

Answer.

(iv). $log0.3109$

Solution.

$$Characteristics = -1$$
$$Mantisa = 0.4926$$
$$Log(0.3109) = \bar{1}.4926$$

Answer.

Question.3. Find the number whose common logarithms are:

(i). 3.5621

Solution.

Since it is log of any number. So,

$$Characteristics = 3$$
$$Mantisa = 0.5621$$
$$Mantisa\ in\ antilog = 3.6484$$

Characteristics change the place of decimal.

So

$$Anti - log(3.5621) = 3648.4$$

Answer.

(ii). $\bar{1}.7427$

Solution.

Since it is log of any number. So,

$$Characteristics = -1$$
$$Mantisa = 0.7427$$
$$Mantisa\ in\ antilog = 5.5297$$

Characteristics change the place of decimal.

So

$$Anti - log(\bar{1}.7427) = 0.5530$$

Answer.

Question.4. what replacement for the unknown in each of the following will make the statement true?

(i). $log_3 81 = L$

Solution.

$$log_3 81 = L$$

Exponential Form

$$3^L = 81$$
$$3^L = 3^4$$
$$=> L = 4.$$

(ii). $log_a 6 = 0.5$

Solution.

$$log_a 6 = 0.5$$

Exponential Form

$$a^{0.5} = 6$$
$$a^{\frac{1}{2}} = 6$$

Squaring on both sides , we have

$$\left(a^{\frac{1}{2}}\right)^2 = 6^2$$
$$a = 36.$$

(iii). $log_5 n = 2$

Solution.

$$log_5 n = 2$$

Exponential Form

$$5^2 = n$$
$$25 = n$$
$$n = 25.$$

(iv). $10^p = 40$

Solution.

$10^p = 40$

Logarithm Form

$$log_{10}40 = p$$
$$p = log_{10}40$$
$$p = 1.6021$$

Question.5. Evaluate

(i). $log_2 \frac{1}{128}$

Solution.

Let

$$x = log_2 \frac{1}{128}$$

Exponential Form

$$2^x = \frac{1}{128}$$
$$2^x = \frac{1}{2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2}$$
$$2^x = 2^{-7}$$
$$=\gg x = -7$$

Answer.

(ii). $log\ 512\ to\ the\ base\ 2\sqrt{2}.$

Solution.

Let

$$x = log_{2\sqrt{2}}512$$

Exponential Form

$$(2\sqrt{2})^x = 512$$

$$\left(2 \times 2^{\frac{1}{2}}\right)^x = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$$

$$\left(2^{1+\frac{1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{2+1}{2}}\right)^x = 2^9$$

$$\left(2^{\frac{3}{2}}\right)^x = 2^9$$

$$2^{\frac{3x}{2}} = 2^9$$

$$==> \frac{3x}{2} = 9$$

$$3x = 18$$

$$x = \frac{18}{3}$$

$$x = 6$$

Answer.

Question.6. Evaluate the value of $''x''$ from the following statements.

(i). $log_2 x = 5$

Solution.

$$log_2 x = 5$$

Exponential Form

$$2^5 = x$$
$$x = 2^5$$

$$x = 2 \times 2 \times 2 \times 2 \times 2$$

$x = 32$**Answer.**

(ii). $log_{81}9 = x$

Solution.

$$log_{81}9 = x$$

Exponential Form

$$81^x = 9$$
$$(9 \times 9)^x = 9$$

$$9^{2x} = 9^1$$

$$==> \quad 2x = 1$$

$$x = \frac{1}{2}$$

Answer.

(iii). $log_{64}8 = \frac{x}{2}$

Solution.

$$log_{64}8 = \frac{x}{2}$$

Exponential Form

$$(64)^{\frac{x}{2}} = 8$$
$$(8 \times 8)^{\frac{x}{2}} = 8$$

$$(8^2)^{\frac{x}{2}} = 8$$

$$8^x = 8^1$$

$$==> \quad x = 1$$

Answer.

(iv). $log_x 64 = 2$

Solution.

$$log_x 64 = 2$$

Exponential Form

$$(x)^2 = 64$$

Taking square root on both sides

$$\sqrt{x^2} = \sqrt{64}$$
$$x = 8$$

Answer.

(v). $log_3 x = 4$

Solution.

$$log_3 x = 4$$

Exponential Form

$$3^4 = x$$
$$x = 3 \times 3 \times 3 \times 3$$
$$x = 81$$

Answer.

Laws of Logarithm

(i) $\log_a(mn) = \log_a m + \log_a n$
(ii) $\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$
(iii) $\log_a m^n = n\log_a m$
(iv) $\log_a n = \log_b n \times \log_a b$

$$or\ = \frac{\log_b n}{\log_b a}$$

(i)

$$\boldsymbol{\log_a(mn) = \log_a m + \log_a n}$$

Proof:

$$Let \log_a m = x\ \ and \log_a n = y$$

Writing in exponential form

$$a^x = m\ \ and\ a^y = n$$
$$\because\ a^x \times a^y = mn$$
$$i.e\ a^{x+y} = mn$$
$$or\ \log_a(mn) = x + y = \log_a m + \log_a n$$
$$hence \log_a(mn) = \log_a m + \log_a n$$

the rule given above is useful in findingthe
Product of two or more numbers using logarithms

Example:
Evaluate $\boldsymbol{291.3 \times 42.36}$
Solution:

$let\ x = 291.3 \times 42.36$
Then$logx = log(291.3 \times 42.36)$

$$= log291.3 + log42.36$$
$$(log_a mn = log_a m + log_a n)$$
$$= 2.4643 + 1.6269 = 4.0912$$
$$x = antilog4.0912 = 12340$$

Example:

$$Evaluate\ 0.2913 \times 0.004236$$

Solution:
Let $y = 0.2913 \times 0.004236$
then$logy = \log 0.2913 + log0.004236$

$$logy = \bar{1}.4643 + \bar{3}.6269$$
$$logy = \bar{3}.0912$$
$$y = antilog\bar{3}.0912$$
$$y = 0.001234$$

$$(ii)\quad \log_a\left(\frac{m}{n}\right) = \log_a m.\log_a n$$

Solution:
Let $log_a m = x\ \ and\ log_a n = y$
So that $a^x = m\ \ and\ a^y = n$

$$\because\ \frac{a^x}{a^y} = \frac{m}{n} \Rightarrow a^{x-y} = \frac{m}{n}$$
$$i.e\quad log_a\left(\frac{m}{n}\right) = x - y = log_a m - log_a n$$

Hence $log_a\left(\frac{m}{n}\right) = log_a m - log_a n$

Note:

(i) $\log_a\left(\frac{m}{n}\right) \neq \frac{\log_a m}{\log_a n}$
(ii) $\log_a m - \log_a n \neq \log_a(m-n)$
(iii) $\log_a\left(\frac{1}{n}\right) = \log_a l - \log_a n = -\log_a n$ ...

$$\because \log_a 1 = 0$$

Note:

(i) $log_a(mn) \neq log_a m \times log_a n$
(ii) $log_a m + log_a n \neq log_a(m+n)$
(iii) $log_a(mnp) = log_a m + log_a n + log_n p + ..$

Example:

$$Evaluate\ \frac{291.3}{42.36}$$
$$let\ \ x = \frac{291.3}{42.36} so\ that\ logx = \log\frac{291.3}{42.36}$$

Then $\log x = log291.33 - log42.36$, ...

$$(\log_a \frac{m}{n} = \log_a m - \log_a n)$$
$$\log x = 2.4643 - 1.6269 = 0{,}8374$$

Thus $x = antilogo.8374 = 6.877$

Example:

$$Evaluate\,\frac{0.0002913}{0.04236}$$

Solution:

$$\boldsymbol{let\ y = \frac{0.0002913}{0.04236}\quad so\ that}$$
$$\boldsymbol{logy = log\left(\frac{0.002913}{0.04236}\right)}$$

Then $\boldsymbol{logy = log\ o.oo2913 - log0.04236}$

$$\boldsymbol{logy = \bar{3}.4643 - \bar{2}.6269}$$
$$\boldsymbol{= \bar{3} + (0.4643 - 0.6269) - \bar{2}}$$
$$\boldsymbol{= \bar{3} - 0.1626 - \bar{2}}$$
$$\boldsymbol{= \bar{3} + (1 - 0.1626) - 1 - \bar{2}}$$

(adding and subtraction 1)

$$\boldsymbol{= \bar{2}.8374}$$
$$\boldsymbol{[\because\ \bar{3} - 1 - \bar{2} = -3 - 1 - (-2) = -2 = \bar{2}]}$$

Therefore , $\boldsymbol{y = antilog\bar{2}.8374}$

$$\boldsymbol{y = 0.06877}$$

(iii) $\boldsymbol{lo_a(m^n) = nlog_a m}$
Proof:

$$\boldsymbol{let\ log_a m^n = x,\quad i.e\ \ a^x = m^n}$$

And$\boldsymbol{log_a m = y,\ i.e\quad a^y = m}$

$$\boldsymbol{then\ a^x = m^n = (a^y)^n}$$
$$\boldsymbol{i.e\ \ a^x = (a^y)^n = a^{yn} \Rightarrow x = ny}$$
$$\boldsymbol{i.e\ \ log_a m^n = nlog_a m}$$

Example:

$$\boldsymbol{Evaluate\ 4\sqrt{(0.0163)^3} = (0.0163)^{\frac{3}{4}}}$$

Solution:

$$\boldsymbol{let\ y = 4\sqrt{(0.0163)^3} = (0.0163)^{\frac{3}{4}}}$$
$$\boldsymbol{loty = \frac{3}{4}(log0.0163)}$$
$$\boldsymbol{= \frac{3}{4} \times \bar{2}.2122}$$
$$\boldsymbol{= \frac{\bar{6}.6366}{4}}$$
$$\boldsymbol{= \frac{\bar{8} + 2.6366}{4}}$$
$$\boldsymbol{= \bar{2} + 0.6592 = \bar{2}.6592}$$

Hence$\boldsymbol{y = antilog\bar{2}.6592}$

$$\boldsymbol{= 0.04562}$$

(iv)Change of base formula:

$$\boldsymbol{log_a n = log_b n \times log_a b\ \ or \frac{log_b n}{log_b a}}$$

Proof:

$$let\ log_b n = x \ \ so\ that\ n = b^x$$

Taking log to the base $a, we\ have$

$$log_a n = log_a b^x = xlog_a b = log_b n log_a b$$

Thus $log_a n = log_b n log_a b \rightarrow (i)$

Putting $n = a\ in\ the\ above\ result, we\ get$

$$log_b a \times log_a b = log_a^a = 1$$

or $log_a b = \frac{1}{log_b a}$

hence equation(i) $gives$

$log_a n = \frac{log_b n}{log_b a} \rightarrow (ii)$

Using the above rule, a natural logarithm can be converted to a common logarithm and vice versa.

$$log_e n = log_{10} n \times log_e 10 \ \ or \frac{log_{10} n}{log_{10} e}$$

$$log_{10} n = log_e n \times log_{10} e \ \ or \frac{log_e n}{log_e 10}$$

The valves of $log_e 10 \ \ and\ log_{10} e \ \ are\ available$

From the tables.

$log_e 10 = \frac{1}{0.4343} = 2.3026$ and

$log_{10} e = log 2.718 = 0.4343$

Example:

Calculate $\boldsymbol{log_2 3 \times log_3 8}$

Solution:

We know that

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\because \ \log_2 3 \times \log_3 8 = \frac{log3}{log2} \times \frac{log8}{log3}$$

$$\frac{log8}{log2} = \frac{log2^3}{log2}$$

$$= \frac{3log2}{log2} = 333$$

Example3.3

. Which of the following into sum of difference.

(i) $\log(A \times \boldsymbol{B})$

Sol: $\log(A \times \boldsymbol{B}) = \log A + \log B$

(ii) $\log\left(\frac{15.2}{30.5}\right)$

Sol: $\log\left(\frac{15.2}{30.5}\right) = \log 15.2 - \log 30.5$

(iii) $\log\left(\frac{21\times5}{8}\right)$

Sol: $\log\left(\frac{21\times5}{8}\right) = \log 21 + \log 5 - \log 8$

(iv) $\log \sqrt[3]{\frac{7}{15}}$

Sol: $\log\left(\frac{7}{15}\right)^{\frac{1}{3}} = \frac{1}{3}\left(\log\frac{7}{15}\right)$

$$= \frac{1}{3}(\log 7 - \log 15)$$

(v) $\log\frac{(22)^{\frac{1}{3}}}{5^3}$

Sol: $\log\frac{(22)^{\frac{1}{3}}}{5^3} = \log(22)^{\frac{1}{3}} - \log 5^3$

$$\log\frac{(22)^{\frac{1}{3}}}{5^3} = \frac{1}{3}\log 22 - 3\log 5$$

(iii) $\log\left(\frac{25\times47}{29}\right)$

Sol: $\log\left(\frac{25\times47}{29}\right) = \log 25 + \log 47 - \log 29$

Q#2) Express $\log x - 2\ \log x + 3\log(x+1) - \log(x^2-1)$ as a single logarithm.

Sol: $\log x - 2\ \log x + 3\log(x+1) - \log(x^2-1)$

$= \log x - \log x^2 + \log(x+1)^3 - \log(x^2-1)$

$= \log\left(\frac{x\ (x+1)^3}{x^2\ (x^2-1)}\right)$

$= \log\left(\frac{(x+1)^3}{x\ (x-1)(x+1)}\right)$

$= \log\left(\frac{(x+1)^2}{x\ (x-1)}\right)$

Q#3) Write the following in the single logarithm.

(i) $\log 21 + \log 5$

Sol: $\log 21 + \log 5 = \log(21 \times \boldsymbol{5})$

(ii) $\log 25 - 2\log 3$

Sol: $\log 2\,5 - 2\log 3 = \log 25 - \log 3^2$

$$= \log\frac{25}{3^2}$$

(iii) $2\log x - 3\log y$

Sol: $2\log x - 3\log y = \log x^2 - \log y^3$

$$= \log\frac{x^2}{y^3}$$

(iv) $\log 5 + \log 6 - \log 2$

Sol: $\log 5 + \log 6 - \log 2 = \log\left(\frac{5\times6}{2}\right)$

Q#4) calculate the following:

(i). $\log_3 2 \times \log_2 81$

Sol: $\log_3 2 \times \log_2 81$

(using $\log_a n = \frac{\log_b n}{\log_b a}$)

$$\log_3 2 \times \log_2 81 = \frac{\log 2}{\log 3} \times \frac{\log 81}{\log 2}$$

$$= \frac{\log 3^4}{\log 3}$$

$$= \frac{4\ \log 3}{\log 3}$$

$$= 4$$

(i). $\log_5 3 \times \log_3 25$

Sol:). $\log_5 3 \times \log_3 25$

(using $\log_a n = \frac{\log_b n}{\log_b a}$)

$$\log_5 3 \times \log_3 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 3}$$

$$= \frac{\log 5^2}{\log 5}$$

$$= \frac{2\ \log 5}{\log 5}$$

$$= 2$$

Q#5) If $\log 2 = 0.3010$, $\log 3 = 0.4171$, and $\log 5 = 0.6990$, then find the values of the following:

(i). $\log 32$

Sol: $\log 32 = \log 2^5 = 5\ \log 2 = 5\ (0.3010)$

$$= 1.5050$$

(ii). $\log 24$

Sol: $\log 24 = \log(2^3 \times 3) = \log 2^3 + \log 3$

$$= 3\ \log 2 + \log 3 = 3\ (0.3010) + (0.4171)$$

$$= 0.9030 + 0.4171 = 1.3801$$

(iii). $\log \sqrt{3\frac{1}{3}}$

Sol: $\log \sqrt{3\frac{1}{3}} = \log \sqrt{\frac{10}{3}} = \log \left(\frac{10}{3}\right)^{\frac{1}{2}}$

$$= \frac{1}{2} \log \frac{10}{3}$$

$$= \frac{1}{2}(\log 10 - \log 3)$$

$$= \frac{1}{2}(\log(2 \times 5) - \log 3)$$

$$= \frac{1}{2}(\log 2 + \log 5 - \log 3)$$

$$= \frac{1}{2}(0.3010 + 0.6990 - 0.4171)$$

$$= \frac{1}{2}(0.5229)$$

$$= 0.2615$$

(iv). $\log \frac{8}{3}$

Sol: $\log \frac{8}{3} = \log 8 - \log 3 = \log 2^3 - log3$

$$= 3\ \log 2 - \log 3 = 3\ (0.3010) - 0.4171$$

$$= 0.9030 - 0.4171$$

$$= 0.4259$$

(v). $\log 30$

Sol: $\log 30 = \log(2 \times 5 \times 3) =$

$$= \log 2 + \log 5 + \log 3$$

$$= 0.3010 + 0.6990 + 0.4171$$

$$= 1.4771$$

Application of logarithm

Example:

Show that

$$\mathbf{7\ \log \frac{16}{15} + 5\ \log \frac{25}{24} + \log \frac{81}{80} = \mathit{log2}}$$

Solution:

$$L.H.S = \boldsymbol{7\ log \frac{16}{15} + 5\ log \frac{25}{24} + log \frac{81}{80}}$$

$$= \boldsymbol{7[log16 - log15] + 5[log25 - log24] + 3[log81 - log80]}$$

$$= \boldsymbol{7[log2^4 - log(3 \times 5)] + 5[log5^2 - log(2^3 \times 3)] + 3[log3^4 - log(2^4 \times 5)]}$$

$$= \boldsymbol{7[4log2 - log3 - log5] + 5[2log5 - 3log2 - log3] + 3[4log3 - 4log2 - log5]}$$

$$= (28 - 15 - 12)log2 + (-7 - 5 + 12)log3 + (-7 + 10 - 3)log5$$

$$= log2 + 0 + 0 = log2 = R.H.S$$

Example

Evaluate $\sqrt[3]{\frac{0.079222 \times (18.99)^2}{(5.79)^4 \times 0.94744}}$

Solution:

Let $y = \sqrt[3]{\frac{0.079222 \times (18.99)^2}{(5.79)^4 \times 0.94744}}$

$$\left(\frac{\mathbf{0.079222} \times (\mathbf{18.99})^{\mathbf{2}}}{(\mathbf{5.79})^{\mathbf{4}} \times \mathbf{0.94744}}\right)^{\frac{1}{3}}$$

$$logy = \frac{1}{3}\log\left(\frac{\mathbf{0.079222} \times (\mathbf{18.99})^{\mathbf{2}}}{(\mathbf{5.79})^{\mathbf{4}} \times \mathbf{0.94744}}\right)$$

$$= \frac{1}{3}[\log\{0.07921 \times (18.99)\} - \log\{(5.79)^2 \times 0.9474\}]$$

$$= \frac{1}{3}[log0.07921 + 2log18.99 - 4log5.79 - log0.9474]$$

$$= \frac{1}{3}[\bar{2}.8988 + 2(1.2786) - 4(0.7627) - \bar{1}.9765]$$

$$= \frac{1}{3}[\bar{2}.8988 + 2.5572 - 3.0508 + 1 - 0.9765]$$

$$= \frac{1}{3}(\bar{2}.4287)$$

$$= \frac{1}{3}(\bar{3} + 1.4287)$$

$$= 1 + 0.4762 = 1.4762$$
$$y = antilog\bar{1}.4762 = 0.299333$$

Example:

Given $A = A_0 e^{-kd}$ if $k = 2$ what should be the Value of d to make $A = \frac{A_0}{2}$?

Solution:

Given that $A = A_0 e^{-kd} \quad \Rightarrow \frac{A}{A_0} = e^{-kd}$

Subtracting $k = 2$ and $A = \frac{A_0}{2}$, we get $\frac{1}{2} = e^{-2d}$

Taking common log on both sides

$$\log_{10} 1 - \log_{10} 2 = -2d\log_{10} e$$

Where $e = 2.718$

$$0 - 0.3010 = -2d(0.4343)$$
$$d = \frac{0.3010}{2 \times 0.4343} = 0.3465$$

Example 3.4

1. Using log tables to find the value of.

(i) 0.8176×13.64

Sol: Let $x = 0.8176 \times 13.64$

Taking log on both sides

$$\log x = \log(0.8176 \times 13.64)$$
$$= \log 0.8176 + \log 13.64$$

(In $\log 0.8176$, the ch. Is $\bar{1}$ we find the log(8.176) which is 0.9125, so combine both that is $\log 0.8176 = \bar{1} + 0.9125 = \bar{1}.9125$)

$$= \bar{1}.9125 + 1.1348$$
$$= -1 + 0.9125 + 1.1348$$
$$= -0.0875 + 1.1348$$
$$\log x = 1.0473$$

Taking anti-log on both sides, we have

$$x = Antilog(1.0473)$$
$$x = 11.15$$

(ii) $(789.5)^{\frac{1}{8}}$

Sol: Let $x = (789.5)^{\frac{1}{8}}$

Taking log on both sides

$$logx = \log(789.5)^{\frac{1}{8}}$$
$$= \frac{1}{8}[\log(789.5)]$$
$$= \frac{1}{8}[2.8974] = \frac{2.8974}{8}$$
$$\log x = 0.3622$$

Taking anti-log on both sides, we have

$$x = Antilog(0.3622)$$
$$x = 2.302$$

(iii) $\frac{0.678 \times 9.01}{0.0234}$

Sol: Let $x = \frac{0.678 \times 9.01}{0.0234}$

Taking log on both sides

$$\log x = \log\left(\frac{0.678 \times 9.01}{0.0234}\right)$$
$$= \log(0.678) + \log(9.01) - \log(0.0234)$$
$$= \bar{1}.8312 + 0.9547 - \bar{2}.3692$$
$$= (-1 + 0.83120) + 0.9547 - (-2 + 0.3692)$$
$$= (-0.1688) + 0.9547 - (-1.6308)$$
$$= -0.1688 + 0.9547 + 1.6308$$
$$= 2.4163$$
$$\log x = 2.4163$$

Taking anti-log on both side, we have

$$x = Antilog(2.4163)$$
$$x = 261$$

(iv) $\sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Sol: Let $x = \sqrt[5]{2.709} \times \sqrt[7]{1.239}$

Taking log on both sides

$$logx = \log(\sqrt[5]{2.709} \times \sqrt[7]{1.239})$$
$$= \log(2.709)^{\frac{1}{5}} + \log(1.239)^{\frac{1}{7}}$$
$$= \frac{1}{5}[\log(2.709)] + \frac{1}{7}[\log(1.239)]$$
$$= \frac{1}{5}[0.4328] + \frac{1}{7}[0.1931]$$
$$= \frac{0.4328}{5} + \frac{0.1931}{7}$$
$$= 0.0866 + 0.0133$$
$$\log x = 0.0999$$

Taking anti-log on both sides, we have

$$x = Antilog(0.0999)$$
$$x = 1.258$$

(v) $\frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Sol: Let $x = \frac{(1.23)(0.6975)}{(0.0075)(1278)}$

Taking log on both sides

$$\log x = \log\left(\frac{(1.23)(0.6975)}{(0.0075)(1278)}\right)$$
$$= \log(1.23) + \log(0.6975) - \log(0.0075) - \log(1279)$$
$$= 0.0899 + \bar{1}.8435 - \bar{3}.8751 - 3.1065$$
$$= 0.0899 + (-1 + 0.8435) - (-3 + 0.8751) - 3.1065$$
$$= 0.0899 + (-0.1565) - (-2.1249) - 3.1065$$
$$= 0.0899 - 0.1565 + 2.1249 - 3.1065$$
$$\log x = -1.0482$$

Adding and subtracting 2 on R.H.S

$$\log x = -2 + 2 - 1.0482$$
$$\log x = \bar{2} + (2 - 1.0482)$$
$$\log x = \bar{2} + (0.9518)$$
$$\log x = \bar{2}.9518$$

Taking anti-log on both side, we have

$$x = Antilog(\bar{2}.9518)$$

(Here $Antilog(0.9518) = 8.50$ but Ch. $\bar{2}$ indicates that point will move two digits to left side)

$$x = 0.0850$$

(iii) $\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

Sol: Let $x = \sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}$

Taking log on both sides

$$\log x = \log\left(\sqrt[3]{\frac{0.7214 \times 20.37}{60.8}}\right)$$
$$= \log\left(\frac{0.7214 \times 20.37}{60.8}\right)^{\frac{1}{3}}$$

$= \frac{1}{3}[\log(0.7214) + \log(20.37) - \log(60.8)]$
$= \frac{1}{3}[\bar{1}.8582 + 1.3090 - 1.7839]$
$= \frac{1}{3}[-1 + 0.8582 + 1.3090 - 1.7839]$
$= \frac{1}{3}[-0.1418 + 1.3090 - 1.7839]$
$= \frac{1}{3}[-0.6157]$
$= -\frac{0.6157}{3}$

$$\log x = -0.2056$$

Adding and subtracting 1 on R.H.S

$$\log x = -1 + 1 - 0.2056$$
$$\log x = \bar{1} + (1 - 0.2056)$$
$$\log x = \bar{1} + (0.7944)$$
$$\log x = \bar{1}.7944$$

Taking anti-log on both side, we have

$$x = Antilog(\bar{1}.7944)$$

(Here $Antilog(0.7944) = 6.229$ but Ch. $\bar{1}$ indicates that point will move one digits to left side)

$$x = 0.6229$$

(v) $\frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Sol: Let $x = \frac{83\times\sqrt[3]{92}}{127\times\sqrt[5]{246}}$

Taking log on both sides

$\log x = \log\left(\frac{83 \times \sqrt[3]{92}}{127 \times \sqrt[5]{246}}\right)$

$= \log(83) + \log(92)^{\frac{1}{3}} - \log(127) - \log(246)^{\frac{1}{5}}$
$= \log(83) + \frac{1}{3}\log(92) - \log(127) - \frac{1}{5}\log(246)$
$= 1.9191 + \frac{1}{3}(1.9638) - 2.1038 - \frac{1}{5}(2.3909)$
$= 1.9191 + \frac{1.9638}{3} - 2.1038 - \frac{2.3909}{5}$
$= 1.9191 + 0.6546 - 2.1038 - 0.4782$

$$\log x = 1.9917$$

Taking anti-log on both side, we have

$$x = Antilog(1.9917)$$
$$x = 0.9811$$

(viii) $\frac{(438)^3\sqrt{0.056}}{(388)^4}$

Sol: Let $x = \frac{(438)^3\sqrt{0.056}}{(388)^4}$

Taking log on both sides

$\log x = \log\left(\frac{(438)^3\sqrt{0.056}}{(388)^4}\right)$

$= \log(438)^3 + \log(0.056)^{\frac{1}{2}} - \log(388)^4$
$= 3\log(438) + \frac{1}{2}\log(0.056) - 4\log(388)$
$= 3(2.6415) + \frac{1}{2}(\bar{2}.7482) - 4(2.5888)$
$= 7.9245 + \frac{1}{2}(-2 + 0.7482) - 10.3552$
$= 7.9245 + \frac{1}{2}(-1.2518) - 10.3552$
$= 7.9245 - 0.6259 - 10.3552$

$$\log x = -3.0566$$

Adding and subtracting 4 on R.H.S

$$\log x = -4 + 4 - 3.0566$$
$$\log x = \bar{4} + (4 - 3.0566)$$
$$\log x = \bar{4} + (0.9434)$$
$$\log x = \bar{4}.9434$$

Taking anti-log on both side, we have

$$x = Antilog(\bar{4}.9434)$$

(Here $Antilog(0.9434) = 8.778$ but Ch. $\bar{4}$ indicates that point will move four digits to left side)

$$x = 0.0008778$$

Q#2) A gas is expanding according to the law $pv^n = C$. Find C, when $p = 80, v = 3.1$ and $n = \frac{5}{4}$.

Sol: $pv^n = C$

Taking log on both sides

$$\log(pv^n) = \log C$$
$$\log C = \log p + \log v^n$$
$$\log C = \log p + n\log v$$

Putting values

$\log C = \log 80 + \frac{5}{4}\log 3.1$
$\log C = 1.9030 + \frac{5}{4}(0.4914)$
$\log C = 1.9030 + 0.6143$
$\log C = 2.5173$

Taking anti-log on both side, we have

$$C = Antilog(2.5173)$$
$$C = 329.2$$

Q#3) The formula $p = 90\,(5)^{-\frac{q}{10}}$ applies to the demand of a product, where q is the number of units and p is the price of one unit. How many units will be demanded if the price is Rs. 18.00?

Sol: $p = 90\,(5)^{-\frac{q}{10}}$

Taking log on both sides

$$\log(p) = \log(90\,(5)^{-\frac{q}{10}})$$
$$\log p = \log 90 + \log((5)^{-\frac{q}{10}})$$
$$\log p = \log 90 - \frac{q}{10}\log 5$$

Putting values

$$\log 18 = \log 90 - \frac{q}{10}\log 5$$
$$1.2553 = 1.9542 - \frac{q}{10}(0.6990)$$
$$1.2553 - 1.9542 = -\frac{q}{10}(0.6990)$$
$$-0.6990 = -\frac{q}{10}(0.6990)$$
$$1 = \frac{q}{10}$$
$$q = 10 \text{ units}$$

Q#4) If $A = \pi r^2$, find A, when $\pi = \frac{22}{7}$ and $r = 15$

Sol: $A = \pi r^2$

Taking log on both sides

$$\log(A) = \log(\pi r^2)$$

$$\log(A) = \log\left(\frac{22r^2}{7}\right)$$

$$\log A = \log 22 + \log(r)^2 - \log 7$$
$$\log A = \log 22 + 2\log r - \log 7$$

Putting values

$$\log A = \log 22 + 2\log 15 - \log 7$$
$$\log A = 1.3424 + 2(1.1761) - 0.8451$$
$$\log A = 1.3424 + 2.3522 - 0.8451$$
$$\log A = 2.8495$$

Taking anti-log on both side, we have

$$A = Antilog(2.8495)$$
$$A = 707.1 \text{ Sq. units}$$

Q#5) If $A = \frac{1}{3}\pi r^2 h$, find A, when $\pi = \frac{22}{7}$, $r = 2.5$ and $h = 4.2$

Sol: $A = \frac{1}{3}\pi r^2 h$

Taking log on both sides

$$\log(A) = \log\left(\frac{1}{3}\pi r^2 h\right)$$
$$\log(A) = \log\left(\frac{22r^2 h}{21}\right)$$
$$\log A = \log 22 + \log(r)^2 + \log h - \log 21$$
$$\log A = \log 22 + 2\log r + \log h - \log 21$$

Putting values

$$\log A = \log 22 + 2\log 2.5 + \log 4.2 - \log 21$$
$$\log A = 1.3424 + 2(0.3979) + 0.6232 - 1.3222$$
$$\log A = 1.3424 + 0.7958 + 0.6232 - 1.3222$$
$$\log A = 1.4392$$

Taking anti-log on both side, we have

$$A = Antilog(1.4392)$$
$$A = 27.49 \text{ cubic units}$$